VIDYA BHAWAN BALIKA VIDYA PITH

शक्तिउत्थानआश्रमलखीसरायबिहार

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10. $f(x) = |x| + |x - 1|$ at $x = 1$

Solution:

Checking the right hand and left hand limits for the given function, we have

$$\lim_{x \to 1^-} f(x) = |x| + |x-1| = \lim_{h \to 0} |1-h| + |1-h-1|$$
$$= |1-0| + |1-0-1| = 1 + 0 = 1$$

$$\lim_{x \to 1^+} f(x) = |x| + |x-1|$$
$$= \lim_{h \to 0} |1+h| + |1+h-1| = 1 + 0 = 1$$

$$\lim_{x \to 1} f(x) = |x| + |x-1| = |1| + |1-1| = 1 + 0 = 1$$

Now, as

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = \lim_{x \to 1} f(x)$$

Thus, f(x) is continuous at x = 1.

Find the value of *k* in each of the Exercises 11 to 14 so that the function *f* is continuous at the indicated point:

11.

$$f(x) = \begin{cases} 3x - 8, & if \;\; x \le 5 \\ 2k \quad , & if \;\; x > 5 \end{cases} \quad \text{at } x = 5$$

Solution:

Finding the left hand and right hand limits for the given function, we have

$$\lim_{x\to 5^-} f(x) = 3x-8$$
$$= \lim_{h\to 0} 3(5-h)-8 = 15-8=7$$
$$\lim_{x\to 5^+} f(x) = 2k$$

As the function is continuous at $x = 5$

$$\lim_{x\to 5^-} f(x) = \lim_{x\to 5^+} f(x)$$

So,

7 = 2k

k = 7/2 = 3.5

Therefore, the value of k is 3.5

12.

$$f(x)=\begin{cases}\dfrac{2^{x+2}-16}{4^x-16}, & if\ x\neq 2\\ k, & if\ x=2\end{cases} \text{ at } \quad x=2$$

Solution:

The given function f(x) can be rewritten as,

$$f(x) = \frac{2^{x+2}-16}{4^x-16} = \frac{2^2.2^x-16}{(2^x)^2-(4)^2} = \frac{4(2^x-4)}{(2^x-4)(2^x+4)}$$
$$f(x) = \frac{4}{2^x+4}$$
$$\lim_{x\to 2^-} f(x) = \lim_{h\to 0}\frac{4}{2^{2-h}+4} = \frac{4}{2^2+4} = \frac{4}{4+4} = \frac{4}{8} = \frac{1}{2}$$
$$\lim_{x\to 2} f(x) = k$$

As the function is continuous at $x = 2$.

$$\therefore \lim_{x\to 2^-} f(x) = \lim_{x\to 2} f(x)$$

So, k = ½

Therefore, the value of k is ½

13.

$$f(x)=\begin{cases} \dfrac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & if\ -1\le x<0 \\ \dfrac{2x+1}{x-1}, & if\ 0\le x\le 1 \end{cases} \quad \text{at } x=0$$

Solution:

Finding the left hand and right hand limits for the given function, we have

$$\lim_{x\to 0^-} f(x) = \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}$$

$$= \lim_{x\to 0^-} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x} \times \frac{\sqrt{1+kx}+\sqrt{1-kx}}{\sqrt{1+kx}+\sqrt{1-kx}}$$

$$= \lim_{x\to 0^-} \frac{(1+kx)-(1-kx)}{x\left[\sqrt{1+kx}+\sqrt{1-kx}\right]}$$

$$= \lim_{x\to 0^-} \frac{1+kx-1+kx}{x\left[\sqrt{1+kx}+\sqrt{1-kx}\right]}$$

$$= \lim_{x\to 0^-} \frac{2kx}{x\left[\sqrt{1+kx}+\sqrt{1-kx}\right]}$$

$$= \lim_{x\to 0^-} \frac{2k}{\sqrt{1+kx}+\sqrt{1-kx}}$$

$$= \lim_{h\to 0} \frac{2k}{\sqrt{1+k(0-h)}+\sqrt{1-k(0-h)}}$$

$$= \frac{2k}{\sqrt{1}+\sqrt{1}} = \frac{2k}{2} = k$$

$$\lim_{x\to 0^+} f(x) = \frac{2x+1}{x-1} = \frac{2(0)+1}{0-1} = \frac{1}{-1} = -1$$

As the function is continuous at $x = 0$.

$$\therefore \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$$

$$k = -1$$

$$\lim_{x \to 0^-} f(x) = \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}$$

$$= \lim_{x \to 0^-} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x} \times \frac{\sqrt{1+kx} + \sqrt{1-kx}}{\sqrt{1+kx} + \sqrt{1-kx}}$$

$$= \lim_{x \to 0^-} \frac{(1+kx) - (1-kx)}{x\left[\sqrt{1+kx} + \sqrt{1-kx}\right]}$$

$$= \lim_{x \to 0^-} \frac{1 + kx - 1 + kx}{x\left[\sqrt{1+kx} + \sqrt{1-kx}\right]}$$

$$= \lim_{x \to 0^-} \frac{2kx}{x\left[\sqrt{1+kx} + \sqrt{1-kx}\right]}$$

$$= \lim_{x \to 0^-} \frac{2k}{\sqrt{1+kx} + \sqrt{1-kx}}$$

$$= \lim_{h \to 0} \frac{2k}{\sqrt{1+k(0-h)} + \sqrt{1-k(0-h)}}$$

$$= \frac{2k}{\sqrt{1} + \sqrt{1}} = \frac{2k}{2} = k$$

$$\lim_{x \to 0^+} f(x) = \frac{2x+1}{x-1} = \frac{2(0)+1}{0-1} = \frac{1}{-1} = -1$$

As the function is continuous at $x = 0$.

$$\therefore \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$$

$$k = -1$$

Therefore, the value of k is -1